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Asymptotic evolution of a passive scalar field advected by an homogeneous turbulent shear flow

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Abstract

The large-time decay of an homogeneous fluctuating scalar field in uniformly sheared homogeneous turbulence is examined following different points of view which are discussed in turn.

Self-preservation analysis of the scalar spectrum equation predicts an exponential decrease of the scalar variance and a constant scalar-to-velocity timescale ratio R. One-point approaches reveal the same qualitative behaviour and the few available experimental data appear to agree with this picture. However, current one-point modelling leads to an asymptotic value of R independently from initial conditions and shear whereas this universality is broken down when allowing for residual vortex stretching in both the velocity and the scalar fields. Further insight into the physics and quantitative evaluation of above concepts would require ad hoc measurements.

1. Introduction

Mean shear, as already known, affects the turbulent diffusion of scalars [1,2]. In addition, mean shear plays on scalar fluctuations variance by causing transfer out of large scales down to smaller ones. This is due to distortion by the mean flow which, similarly, results in transfer of velocity fluctuations energy [3,4]. In the equation for the mean destruction rate of scalar fluctuations (the so-called scalar dissipation), this mechanism gives rise to a production term including explicitly mean velocity gradients [5]. Whereas mean scalar gradients sustain production of both scalar fluctuations energy and scalar dissipation, mean shear exclusively promotes this latter. Thus, in case of vanishing scalar gradients, the fluctuating scalar field nevertheless interacts with the mean flow via mean shear.

Modelling and theoretical works [6–8] as well as experimental investigations on laboratory flows [9] and on dispersion in the atmospheric surface layer [10] have established the shear-induced enhancement of the destruction rate of scalar fluctuations. In particular, it has been shown that mean shear can act significantly on concentration fluctuations decay in the far field of sources [6,7]. Still, thorough investigations on the sole influence of shear are somewhat lacking for existing studies often gather the respective contributions of scalar and velocity mean gradients together [2,11–13].

In the present work, the decay of a zero-mean-gradient fluctuating passive scalar field in a shear flow is analyzed with intent to bring out the effect of shear. This situation may be related, for instance, to the far field of a concentrated source of contaminant in which the concentration gradients vanish while the effect of shear increases as the lengthscales of the concentration field grow. Furthermore, an uniform passive scalar field (say, temperature) submitted to a controlled mean shear can be available in the laboratory by combining a slightly heated grid with a shear generator [14].

It will be assumed that the flow is represented by homogeneous sheared turbulence which retains most features of inhomogeneous turbulence while relieving

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Nomenclature

b_{ij}	anisotropy tensor components, $(\overline{u_i'u_j'} - \frac{1}{3}q^2\delta_{ij})/q^2$
C_D, C_S	constants of modelled dissipation and stretching terms of scalar variance dissipation
C_{s_θ}	constant of the low-wavenumber power-law of the scalar spectrum
C_μ, C_{ϵ_1}, C_{ϵ_2}	constants of the standard $k-\epsilon$ model
$C_{\nabla u}$	constant of modelled shear-production term of scalar variance dissipation
D_θ	molecular diffusivity of scalar θ
E_θ	scalar spectrum
k	wavenumber modulus
K	turbulent kinetic energy
L_θ	similarity lengthscale of the scalar field
P	production of turbulent kinetic energy
$P_{\nabla u}$	production of scalar variance dissipation by mean shear
Pe_λ	Péclet number, $q\lambda_\theta/D_\theta$
q^2	trace of Reynolds stress tensor, $q^2 = 2$ K
R	scalar-to-velocity timescale ratio, $\overline{\theta'^2}\epsilon/q^2\epsilon_\theta$
Re_t	turbulence Reynolds number, $K^2/\nu\epsilon$
s_θ	exponent of low-wavenumber law of the scalar spectrum
S	mean shear
$S\mathscr{E}_\theta$	spectral transfer due to mean shear
Sc	Schmidt number, ν/D_θ
T_θ	spectral transfer by nonlinear interactions
t	time
u_i	velocity components
x_i	space coordinates

Greek symbols

β	growth rate of turbulent kinetic energy
β_θ	decay rate of scalar variance
δ_{ij}	Kronecker's tensor
ϵ	dissipation of turbulent kinetic energy
ϵ_θ	scalar variance dissipation
η_C	Corrsin's microscale, $(D_\theta^3/\epsilon)^{1/4}$
η_2, η_3	small parameters
λ_θ	scalar field microscale
Λ_θ	scalar field integral lengthscale
ν	kinematic viscosity
θ	passive scalar
τ_t	turbulent kinetic energy dissipation timescale, $q^2/2\epsilon$
τ_θ	scalar variance dissipation timescale, $\overline{\theta'^2}/2\epsilon_\theta$
ψ_ϵ	dimensionless decay rate of turbulent kinetic energy dissipation
$\boldsymbol{\psi_\theta}$	dimensionless decay rate of scalar variance dissipation
ζ_1, ζ_2, ζ_3	small parameters

Subscript

∞	asymptotic value

the treatment of complications such as wall boundaries and turbulent transport. Although inconsistencies inherent in this concept have early been pointed out, [4,15–17] experimental studies on nearly homogeneous shear flows have been reported by Rose [18], Champagne et al. [15], Mulhearn and Luxton [19], Harris et al. [16], Tavoularis and Corrsin [11,12] and Tavoularis and Karnik [20], among others. Theoretical analysis [17] suggests that an homogeneous shear flow may reach a self-similar regime in which the turbulent kinetic energy K and its dissipation rate ϵ grow exponentially at the same rate while the anisotropy tensor as well as the dimensionless timescale SK/ϵ (where S is the uniform mean shear) become constant independently from initial conditions. This picture is confirmed by available experimental data [20,21] and is in agreement with current modelling of turbulence [22].

Recently, George and Gibson [23] have examined self-similar solutions of the energy spectrum equation in homogeneous sheared turbulence. Their analysis shows the existence of a self-similarity regime in which both the Taylor microscale and the integral lengthscale are constant. They also find K and ϵ to increase exponentially and SK/ϵ and the anisotropy tensor to be constant. George and Gibson [23] stress, in addition, that although the energy spectrum reaches a self-similarity regime, its shape is determined by initial conditions. In the field of one-point modelling, the model proposed by Bernard and Speziale [24] departs from the current $K-\epsilon$ concept in that it accounts for residual vortex stretching. According to this approach, the dynamic field asymptotically tends to a production-equals-dissipation regime in which K and ϵ attain constant values determined by initial conditions. The dimensionless timescale SK/ϵ and the anisotropy tensor reach an universal equilibrium while the turbulence Reynolds number assumes a constant value depending on the residual vortex stretching. The authors show this regime to occur at rather large total strain that is, $St > 30$ which makes the experimental verification of their analysis difficult at present.

The objective of the study is to draw the implications of different concepts regarding the effect of mean shear on scalar fluctuations. To this aim, three approaches are successively analyzed and compared. In Section 2, the asymptotic behaviour of the scalar field is derived from a self-similarity analysis of the scalar variance spectrum equation. Section 3 is devoted to one-point approaches. Firstly, the large-time decay of scalar fluctuations in the presence of mean shear is examined in the context of current modelling. Afterwards, the alternative point of view of Bernard and Speziale [24] which, up to the present, has not been fully validated, is extended to the scalar dissipation. A somewhat richer picture of the asymptotics of the scalar field arises from accounting for the effect of residual vortex stretching. Qualitative comparisons with some experimental data are discussed in section 4.

2. Spectral self-preservation analysis

Self-preserving solutions in the presence of shear can be derived from the spectral equation as already done for the velocity field [23]. The equation for the three-dimensional spectrum of scalar fluctuations is obtained following standard techniques [4] that is, applying Fourier transform to the two-point correlation equation. Assuming zero mean scalar gradient, homogeneous turbulence and a mean velocity field $\overline{u_i} = Sx_2\delta_{i1}$ with uniform and constant shear S, the scalar spectral equation is finally written:

$$\frac{\partial E_\theta(k,t)}{\partial t} - S\mathscr{E}_\theta(k,t) = T_\theta(k,t) - 2D_\theta k^2 E_\theta(k,t). \qquad (1)$$

The wavenumber vector is denoted by $\vec{k}$ with $|\vec{k}| = k_i k_i$ ($i = 1, 2, 3$). All quantities of Eq. (1) represent averages over spherical shells of radius k:

$$E_\theta(k,t) = 2\pi k^2 [E_{\theta,\theta}(\vec{k},t)]_{\text{av}}$$

$$T_\theta(k,t) = 2\pi k^2 [T_{\theta,\theta}(\vec{k},t)]_{\text{av}}$$

$$\mathscr{E}_\theta(k,t) = 2\pi k^2 \left[k_1 \frac{\partial E_{\theta,\theta}(\vec{k},t)}{\partial k_2}\right]_{\text{av}}$$

with:

$$[E_{\theta,\theta}(\vec{k},t)]_{\text{av}} = \frac{1}{4\pi k^2}\iint_{k=|\vec{k}|} E_{\theta,\theta}(\vec{k},t)\,\mathrm{d}\mathscr{A}(k)$$

$$[T_{\theta,\theta}(\vec{k},t)]_{\text{av}} = \frac{1}{4\pi k^2}\iint_{k=|\vec{k}|} T_{\theta,\theta}(\vec{k},t)\,\mathrm{d}\mathscr{A}(k)$$

$$\left[k_1 \frac{\partial E_{\theta,\theta}(\vec{k},t)}{\partial k_2}\right]_{\text{av}} = \frac{1}{4\pi k^2}\iint_{k=|\vec{k}|} k_1 \frac{\partial E_{\theta,\theta}(\vec{k},t)}{\partial k_2}\,\mathrm{d}\mathscr{A}(k).$$

$E_{\theta,\theta}(\vec{k},\ t)$ is the Fourier transform of the two-point scalar correlation and $T_{\theta,\theta}(\vec{k},\ t)$ is related to the Fourier transforms of the two-point velocity-scalar triple correlations.

Terms on the right-hand side of Eq. (1) are usual in shear-free turbulence and represent, respectively, spectral transfer by nonlinear interactions and molecular dissipation; in the last term, D_θ is the molecular diffusivity of the scalar quantity θ. The second term on the left-hand side of the spectral equation arises from the presence of mean shear and represents spectral transfer due to distortion by the mean flow. Mean shear causes also transfer through the velocity fluctuations spectrum. This was analyzed in detail by Lumley [3] (see also Ref. [4]) who showed that this mechanism is confined into the anisotropic low-wavenumber range, extracting energy from large scales and feeding the smaller ones. It is to be reminded that for both transfer terms of the spectral equation:

$$\int_0^\infty T_\theta(k,t)\,\mathrm{d}k = 0 \quad \text{and} \quad \int_0^\infty S\mathscr{E}_\theta(k,t)\,\mathrm{d}k = 0.$$

Mean shear thus does not affect the total variance budget but hastens the decay via enhancement of spectral transfer.

Now, looking for self-similar solutions of Eq. (1), let us write:

$$E_\theta(k,t) = \tilde{E}_\theta(t) f_{\text{E}}(\tilde{k})$$

$$T_\theta(k,t) = \tilde{T}_\theta(t) f_{\text{T}}(\tilde{k})$$

$$S\mathscr{E}_\theta(k,t) = S\tilde{\mathscr{E}}_\theta(t) f_{\mathscr{E}}(\tilde{k})$$

with $\tilde{k} = kL_\theta(t)$. Reporting above expressions in Eq. (1) and multiplying all terms by $L_\theta^2(t)/D_\theta\tilde{E}_\theta(t)$ yields:

$$\frac{L_\theta^2(t)}{D_\theta\tilde{E}_\theta(t)}\frac{\mathrm{d}\tilde{E}_\theta}{\mathrm{d}t} f_{\text{E}}(\tilde{k}) + \frac{L_\theta(t)}{D_\theta}\frac{\mathrm{d}L_\theta}{\mathrm{d}t}\tilde{k}\frac{\mathrm{d}f_{\text{E}}}{\mathrm{d}\tilde{k}}$$
$$= \frac{L_\theta^2(t)}{D_\theta\tilde{E}_\theta(t)}\tilde{T}_\theta(t) f_{\text{T}}(\tilde{k}) + \frac{L_\theta^2(t)}{D_\theta\tilde{E}_\theta(t)} S\tilde{\mathscr{E}}_\theta(t) f_{\mathscr{E}}(\tilde{k})$$
$$- 2\tilde{k}^2 f_{\text{E}}(\tilde{k}).$$

Existence of self-similar solutions requires:

$$\frac{L_\theta^2(t)}{D_\theta \tilde{E}_\theta(t)} \frac{d\tilde{E}_\theta}{dt} = \text{constant} \tag{2}$$

$$\frac{L_\theta(t)}{D_\theta} \frac{dL_\theta}{dt} = \text{constant} \tag{3}$$

$$\frac{L_\theta^2(t)\tilde{T}_\theta(t)}{D_\theta \tilde{E}_\theta(t)} = \text{constant} \tag{4}$$

$$\frac{L_\theta^2(t) S\tilde{\mathscr{E}}_\theta(t)}{D_\theta \tilde{E}_\theta(t)} = \text{constant.} \tag{5}$$

Besides, definition of the total variance

$$\overline{\theta'^2}(t) = 2\int_0^\infty E_\theta(k, t)\,dk$$

yields $\tilde{E}_\theta(t) \propto \overline{\theta'^2}(t) L_\theta(t)$ which, with Eqs. (4) and (5), leads to:

$$\tilde{T}_\theta(t) \propto S\tilde{\mathscr{E}}_\theta(t) \propto D_\theta \overline{\theta'^2}(t)/L_\theta(t).$$

In addition, Eq. (3) gives: $L_\theta^2(t) = aD_\theta t + b$. In Eq. (5), this expression implies $S\tilde{\mathscr{E}}_\theta(t)/\tilde{E}_\theta(t) \sim t^{-1}$ (if $a \neq 0$). Since $\tilde{\mathscr{E}}_\theta$ and $\tilde{E}_\theta$ are both related to the spectrum $E_{\theta,\theta}$, self-preservation requires $\tilde{\mathscr{E}}_\theta(t) \propto \tilde{E}_\theta(t)$ and, hence, $S \sim t^{-1}$. This solution arises also from the study of the equation for the velocity fluctuations spectrum [23]. As the analysis is restricted to constant shear, we necessarily have $a = 0$ and, therefore, L_θ=constant. The definition of the mean destruction rate of scalar fluctuations

$$\epsilon_\theta(t) = 2D_\theta \int_0^\infty k^2 E_\theta(k, t)\,dk$$

gives, with the previous relations:

$$\epsilon_\theta(t) \propto D_\theta \overline{\theta'^2}(t)/L_\theta^2$$

which suggests $L_\theta \propto \lambda_\theta$, since $\epsilon_\theta(t) = 6CD_\theta \overline{\theta'^2}(t)/\lambda_\theta^2$ with λ_θ being the scalar microscale and C a constant accounting for anisotropy. This latter result implies, using Eq. (2):

$$\tilde{E}_\theta(t) = \tilde{E}_\theta(0) \exp(\beta_\theta D_\theta t/\lambda_\theta^2)$$

with β_θ=constant. One can also write:

$$\overline{\theta'^2}(t) = \overline{\theta'^2}(0) \exp(\beta'_\theta St)$$

with $\beta'_\theta = \beta_\theta D_\theta/\lambda_\theta^2 S$. As a consequence:

$$\frac{d\overline{\theta'^2}}{dt} = S\beta'_\theta \overline{\theta'^2} = -2\epsilon_\theta$$

and hence: $\beta'_\theta = -(S\tau_\theta)^{-1}$ where $\tau_\theta = \overline{\theta'^2}/2\epsilon_\theta$, the scalar timescale, is constant. It is also straightforward to show that the scalar integral lengthscale:

$$\Lambda_\theta = \frac{\pi}{\overline{\theta'^2}} \int_0^\infty E_\theta(k, t) k^{-1}\,dk$$

is proportional to L_θ and is thereby constant.

Finally, the above analysis suggests that during self-similar decay of scalar fluctuations in homogeneous sheared turbulence, both the variance $\overline{\theta'^2}$ and the destruction rate ϵ_θ decrease exponentially, at the same rate, keeping the timescale, the integral lengthscale and the microscale constant. This, of course, sharply contrasts with self-preserving decay in homogeneous, shearless turbulence during which $\overline{\theta'^2}$ and ϵ_θ follow power laws and all scales grow with time [4]. Moreover, since in the case of homogeneous shear the approach of George and Gibson [23] leads to exponential increase of both the energy of turbulence $q^2/2$ and the dissipation rate ϵ with $\tau_t = q^2/2\epsilon$ remaining constant, it can also be inferred that the Corrsin's microscale $\eta_C = (D_\theta^3/\epsilon)^{1/4}$ decreases exponentially while the Péclet number $Pe_\lambda = q\lambda_\theta/D_\theta$ increases and the scalar-to-velocity timescale ratio $R = \tau_\theta/\tau_t$ reaches a constant value. It is also worth noticing that the lengthscale defined as $l_\theta = q\tau_\theta$ increases exponentially.

Self-preservation has further consequences on the shape of the spectrum. At low wavenumbers, assuming a power law, the scalar variance spectrum can be written, in the self-similar form:

$$\frac{E_\theta(k, t)}{\overline{\theta'^2}\lambda_\theta} = \frac{C_{s_\theta}}{\overline{\theta'^2}\lambda_\theta^{s_\theta+1}} (k\lambda_\theta)^{s_\theta} \quad \text{for} \quad k \to 0.$$

Since $\overline{\theta'^2}$ and λ_θ are the similarity variables, the ratio $C_{s_\theta}/\overline{\theta'^2}\lambda^{s_\theta+1}$ is a constant if self-preservation is valid in this spectral range. Then, the constancy of λ_θ and the exponential time evolution of $\overline{\theta'^2}$ imply that C_{s_θ} cannot be constant but decreases in time as $C_{s_\theta} \sim \exp(-\beta'_\theta St)$. The inertial range, if any, is defined, in terms of self-similarity variables, by:

$$\frac{E_\theta(k, t)}{\overline{\theta'^2}\lambda_\theta} = C_\theta(\epsilon_\theta/\overline{\theta'^2})\epsilon^{-1/3}\lambda_\theta^{2/3}(k\lambda_\theta)^{-5/3}.$$

As $\epsilon_\theta/\overline{\theta'^2}$ and λ_θ are constant in the self-preserving regime, we necessarily have $C_\theta \sim \exp(\beta t/3)$ where β is the growth rate of q^2 and ϵ; this also means that $C_\theta \propto Re_t^{1/3}$, Re_t being the turbulence Reynolds number. Note, finally, that a k^{-1} scaling of the scalar variance spectrum in the form $E_\theta(k, t) = C_1\epsilon_\theta T k^{-1}$, with T being a constant timescale, implies:

$$\frac{E_\theta(k, t)}{\overline{\theta'^2}\lambda_\theta} = \frac{1}{2} C_1 T \tau_\theta^{-1} (k\lambda_\theta)^{-1}$$

in which, this time, C_1 is a constant since in the self-preserving regime τ_θ is constant. In the special case $T=q^2/2\epsilon$ (which is constant during self-preservation), we have:

$$\frac{E_\theta(k,t)}{\overline{\theta'^2}\lambda_\theta}=C_1R^{-1}(k\lambda_\theta)^{-1}.$$

3. One-point approaches

3.1. Current modelling of homogeneous shear turbulence

In an homogeneous turbulent shear flow the mean velocity of which is expressed as $\overline{u_i}=Sx_2\delta_{i1}$ where S is the constant and uniform mean shear, the evolution of q^2 is given by the exact equation:

$$\frac{dq^2}{dt}=-2\overline{u_1'u_2'}S-2\epsilon.$$

The most usual modelling of the ϵ-equation is [24]:

$$\frac{d\epsilon}{dt}=-\psi_\epsilon\frac{\epsilon^2}{q^2}$$

with $\psi_\epsilon=2C_{\epsilon_1}\overline{u_1'u_2'}S/\epsilon+2C_{\epsilon_2}$, where C_{ϵ_1} and C_{ϵ_2} are model constants. A $K-\epsilon$ type modelling of the Reynolds stress tensor yields $\overline{u_1'u_2'}=-C_\mu(q^4/4\epsilon)S$, C_μ being a constant. In the framework of this classical modelling, it is easy to prove that the turbulence timescale, the component b_{12} of the anisotropy tensor and the production-to-dissipation ratio tend to asymptotic values, respectively, $(q^2/2\epsilon)_\infty$, $(\overline{u_1'u_2'}/q^2)_\infty$ and $(-\overline{u_1'u_2'}S/\epsilon)_\infty$ which are independent from initial conditions. The kinetic energy of turbulence and its mean dissipation rate increase exponentially in time at the rate $\beta=2[(\epsilon/P)_\infty-1](\overline{u_1'u_2'}/q^2)_\infty S$ (P representing production) and so does the turbulence Reynolds number.

In this type of flow, fluctuations of an homogeneous scalar field decay according to:

$$\frac{d\overline{\theta'^2}}{dt}=-2\epsilon_\theta \tag{6}$$

$$\frac{d\epsilon_\theta}{dt}=-\psi_\theta\frac{\epsilon_\theta^2}{\overline{\theta'^2}}. \tag{7}$$

Eq. (7) is nothing but a compact form of the ϵ_θ-equation written by analogy with the above ϵ-equation using the dimensionless decay rate ψ_θ which is to be modelled [32]. Terms requiring closure are vortex stretching, molecular dissipation and mean shear. In the exact equation for ϵ_θ, this latter is represented by the term [5]: $-2D_\theta\overline{\partial\theta'/\partial x_\alpha.\partial\theta'/\partial x_\beta}.\partial\overline{u_\alpha}/\partial x_\beta$ which, in the present case, is reduced to $-2D_\theta\overline{\partial\theta'/\partial x_1.\partial\theta'/\partial x_2}.S$. Standard models [25–27] represent production of ϵ_θ by mean shear, $P_{\nabla u}$, as:

$$P_{\nabla u}=-2C_{\nabla u}b_{12}S\epsilon_\theta \quad \text{with} \quad C_{\nabla u}=\text{constant}.$$

Hence, ψ_θ can be written as:

$$\psi_\theta=\psi_\theta^{(m)}+4C_{\nabla u}b_{12}S(q^2/2\epsilon)R$$

where $R=\overline{\theta'^2}\epsilon/q^2\epsilon_\theta$ is the scalar-to-velocity timescale ratio.

Modelled equations for ϵ_θ differ from each other regarding the expression for $\psi_\theta^{(m)}$ that is, the modelling of stretching and molecular dissipation. The antagonistic effects of these mechanisms in the budget of ϵ_θ, the former acting as production and the latter as destruction, have early been recognized [28–30]. Zeman and Lumley [31], however, modelled the sum of these terms as a destruction mechanism. The model of Newman et al. [32] is similar in this respect and is written:

$$\psi_\theta^{(m)}=2C_SR+C_D.$$

C_S and C_D are constants. This model was implemented in a number of subsequent numerical studies [25–27,33,34]. On the other hand, modelling stretching and molecular dissipation as opposed mechanisms was proposed by Lumley and Khajeh-Nouri [28]. Mantel and Borghi [35] recently adopted the same view and, furthermore, argued for the necessity to include the turbulence Reynolds number in the modelling of $\psi_\theta^{(m)}$ in order to agree with the order-of-magnitude analysis [28,29,35,36]. Their model is expressed as:

$$\psi_\theta^{(m)}=-Re_t^{1/2}(2C_SR-C_D).$$

Note that the same notations are used for the constants although they may assume distinct values in each model.

The asymptotic decay of scalar fluctuations given by the above models is examined assuming that the dynamic field has reached equilibrium that is, the turbulence timescale and b_{12} are constant and equal, respectively, to:

$$(q^2/2\epsilon)_\infty=[(C_{\epsilon_2}-1)/C_\mu(C_{\epsilon_1}-1)]^{1/2}S^{-1}$$

$$(b_{12})_\infty=-\frac{1}{2}[C_\mu(C_{\epsilon_2}-1)/(C_{\epsilon_1}-1)]^{1/2}.$$

From Eqs. (6) and (7), the equation for R is easily derived:

$$\frac{dR}{dt}=\frac{1}{2}(\psi_\theta-2)\left(\frac{2\epsilon}{q^2}\right)_\infty.$$

An equilibrium state is reached when R equals R_∞ in such a way that $\psi_\theta=2$ which yields, with a Newman et al. [32] type modelling of $\psi_\theta^{(m)}$:

$$R_\infty = \frac{2 - C_D}{2[C_S + 2C_{\nabla u}(b_{12})_\infty S(q^2/2\epsilon)_\infty]}.$$

R_∞ is a positive non-zero value if $C_D > 2$ and $C_S/C_{\nabla u} < -2(b_{12})_\infty S(q^2/2\epsilon)_\infty$ or $C_D < 2$ and $C_S/C_{\nabla u} > -2(b_{12})_\infty S(q^2/2\epsilon)_\infty$. According to the above equations for $(b_{12})_\infty$ and $(q^2/2\epsilon)_\infty$,

$$2C_{\nabla u}(b_{12})_\infty S(q^2/2\epsilon)_\infty = -C_{\nabla u}(C_{\epsilon_2} - 1)/(C_{\epsilon_1} - 1)$$

and R_∞ is consequently independent from initial conditions. With usual values of C_{ϵ_1} and C_{ϵ_2} ($C_{\epsilon_1} \simeq 1.45$, $C_{\epsilon_2} \simeq 1.9$), the ratio $(C_{\epsilon_2}-1)/(C_{\epsilon_1}-1)$ is close to 2.

The constraint $C_D \geq 2$ satisfies the requirement imposed by Newman et al. [32] that, in homogeneous decaying turbulence, R should reach its equilibrium value monotonically. In some models [25,37] the value $C_D = 2$ is chosen in order to ensure that, in freely decaying turbulence, $dR/dt = 0$. However, in the present case of zero mean scalar gradient with shear, $C_D = 2$ would lead to $R_\infty = 0$. This value is not unphysical but represents a limit in which the scalar variance decays exponentially at an infinite rate implying that fluctuations completely vanish. Models with $C_D > 2$ and $C_S/C_{\nabla u} < 2$ ensure that R_∞ is nonzero and positive. This is realized, for instance, by the models of Yoshizawa [26] and Nagano and Kim [27] in which $(C_D, C_S, C_{\nabla u}) = (2.4, 0.52, 0.52)$ and $(2.2, 0.8, 0.72)$ yield $R_\infty = 0.38$ and $R_\infty = 0.16$, respectively. It is also straightforward to show that, in both models, R_∞ is a stable fixed point.

In the framework of the model of Mantel and Borghi [35], the equilibrium state is defined by:

$$R_\infty = \frac{-C_D Re_t^{1/2} + 2}{2[-C_S Re_t^{1/2} + 2C_{\nabla u}(b_{12})_\infty S(q^2/2\epsilon)_\infty]}.$$

As reported previously, $K-\epsilon$ type modelling of homogeneous turbulent shear flows leads to unbounded growth of the turbulence Reynolds number with time, in agreement with the analysis of Tavoularis [17] and the recent study of George and Gibson [23]. In the model of Mantel and Borghi, the equilibrium for the scalar field is thereby reached asymptotically as Re_t tends to infinity which imposes R tending to $R_\infty = C_D/2C_S$. This limit is free from shear-induced effect since the $C_{\nabla u}$-term becomes vanishingly small in comparison with the $Re_t^{1/2}$-term. This models consequently ignores the effect of mean gradients on ϵ_θ when applied to large Reynolds number turbulent shear flows [38]. Despite their basic differences, both Newman et al. and Mantel and Borghi models, in the case under study, yield an asymptotic regime in which $\psi_\theta = 2$ and θ'^2 decays exponentially at a rate equal to $\tau_{\theta_\infty}^{-1} = [R_\infty(q^2/2\epsilon)_\infty]^{-1}$.

3.2. Alternative modelling and extension to the scalar field

Analyzing the self-preserving regime of velocity fluctuations decay, Speziale and Bernard [39] have proved that accounting for momentary unbalanced vortex stretching provides a more complete understanding of isotropic turbulence than current modelling does. This approach has recently been extended to the case of a scalar field and validated by means of comparison with experimental data on temperature fluctuations decay in grid turbulence [40]. Bernard and Speziale [24] call upon the same concept in their study of homogeneous shear turbulence and, allowing for small departures from equilibrium, derive a model which, at large times, yields an asymptotic production-equals-dissipation regime with bounded energy and dissipation. Interestingly, this contrasts with the asymptotic trend of standard models. It is also worth noticing that Tavoularis and Karnik [20] showed that an asymptotic regime with constant kinetic energy can be compatible with their measurements in the case of low shear. The experimental investigation of the large-time evolution of homogeneous shear turbulence, however, is delicate and the question of a possible bounded-energy state is still an open question. Keeping this limitation in mind, the approach of Bernard and Speziale [24] is analyzed with regard to the effect of mean shear on the destruction rate of scalar fluctuations.

In the framework of this model, the nondimensional rate of change of ϵ is:

$$\psi_\epsilon = 2C_{\epsilon_1}\overline{u_1'u_2'}\frac{S}{\epsilon} - \frac{14}{3\sqrt{15}}C_{\epsilon_3}\frac{q^2}{2(\nu\epsilon)^{1/2}} + 2C_{\epsilon_2}$$

with

$$C_{\epsilon_2} = 2 - \eta_2 \quad \text{and} \quad C_{\epsilon_3} = \frac{3\sqrt{15}}{14}\eta_3.$$

η_2 and η_3 are small parameters accounting for departure from the regime in which there is no residual vortex stretching. The second term on the right-hand side corresponds to residual vortex stretching and the current model for ψ_ϵ is retrieved with $C_{\epsilon_3} = 0$. With $K-\epsilon$ modelling of the Reynolds stress tensor, this approach produces equilibrium states in which q^2 and ϵ are constant. The turbulence timescale, the b_{12} component of the anisotropy tensor as well as the turbulence Reynolds number tend to the following constant values:

$$(q^2/2\epsilon)_\infty = 1/S\sqrt{C_\mu}$$

$$(b_{12})_\infty = -\sqrt{C_\mu}/2$$

$$Re_{t_\infty} = \frac{135}{49}\left[\frac{C_{\epsilon_2} - C_{\epsilon_1}}{C_{\epsilon_3}}\right]^2.$$

Note that $(q^2/2\epsilon)_\infty$ and $(b_{12})_\infty$ differ from those obtained in current models (Section 3.1). It is to be noted, however, that the numerical results of Bernard and Speziale agree with those from standard models for elapsed time $St < 30$.

A model accounting for non-equilibrium due to initial unbalanced vortex stretching has been proposed for addressing the approach to self-preservation of an homogeneous scalar field [40]. From the exact ϵ_θ-equation, $\psi_\theta^{(m)}$ has been written as follows:

$$\psi_\theta^{(m)} = -\frac{5\sqrt{2}}{9\sqrt{3}} S_\theta Pe_\lambda + \frac{10}{9} G_\theta.$$

S_θ is related to the mixed-derivative skewness coefficient and G_θ is the coefficient of scalar enstrophy destruction. Then, the expression characterizing complete self-preserving decay has been derived to be, at large Reynolds and Péclet numbers:

$$\frac{10}{9} G_\theta = \frac{5\sqrt{2}}{9\sqrt{3}} S_\theta Pe_\lambda + 2R + 2.$$

Now, similarly to the study of Bernard and Speziale [24] for the velocity field, small departures from the above equilibrium regime are expressed as:

$$\frac{10}{9} G_\theta = \left(\frac{5\sqrt{2}}{9\sqrt{3}} S_\theta - \zeta_3\right) Pe_\lambda + (2 - \zeta_1)R + (2 - \zeta_2)$$

where ζ_1, ζ_2 and ζ_3 are small parameters.

Consequently, using $Pe_\lambda^2 = 24 Sc R Re_t$, $\psi_\theta^{(m)}$ becomes:

$$\psi_\theta^{(m)} = C_{\epsilon_{\theta_1}} R + C_{\epsilon_{\theta_2}} - \frac{20}{9} C_{\epsilon_{\theta_3}} Sc^{1/2} Re_t^{1/2} R^{1/2}$$

with

$$C_{\epsilon_{\theta_1}} = 2 - \zeta_1; \quad C_{\epsilon_{\theta_2}} = 2 - \zeta_2; \quad C_{\epsilon_{\theta_3}} = \frac{9\sqrt{3}}{5\sqrt{2}} \zeta_3.$$

Sc is the Schmidt number, $Sc = \nu / D_\theta$. The above $\psi_\theta^{(m)}$ corresponds to a generalized Newman et al. [32] type model (section 3.1.) including a non-equilibrium $(Re_t R)^{1/2}$-term resulting from residual vortex stretching. Then, accounting for homogeneous shear, ψ_θ is written as follows:

$$\psi_\theta = 4 C_{\nabla u} b_{12} S \frac{q^2}{2\epsilon} + \psi_\theta^{(m)}$$

that is,

$$\psi_\theta = \left[4 C_{\nabla u} b_{12} S \frac{q^2}{2\epsilon} + C_{\epsilon_{\theta_1}}\right] R + C_{\epsilon_{\theta_2}} - \frac{20}{9} C_{\epsilon_{\theta_3}} Sc^{1/2} Re_t^{1/2} R^{1/2}.$$

As previously, it is assumed that the scalar field evolves in a dynamic field which has already reached the asymptotic state and hence, in the framework of the model of Bernard and Speziale, $q^2/2\epsilon$, b_{12} and Re_t are constant. Using their asymptotic expressions in ψ_θ yields:

$$\psi_\theta = (C_{\epsilon_{\theta_1}} - 2C_{\nabla u})R + C_{\epsilon_{\theta_2}} - \frac{20\sqrt{5}}{7\sqrt{3}} (C_{\epsilon_2} - C_{\epsilon_1}) \times \left(\frac{C_{\epsilon_{\theta_3}}}{C_{\epsilon_3}}\right) Sc^{1/2} R^{1/2}. \quad (8)$$

The non-equilibrium term includes the effects of residual vortex stretching on both the dynamic and the scalar fields through the ratio $C_{\epsilon_{\theta_3}}/C_{\epsilon_3}$.

The fixed-point equation for R, resulting from $\psi_\theta = 2$ (Section 3.1), is:

$$aR_\infty + bR_\infty^{1/2} + c = 0 \quad (9)$$

with

$$a = C_{\epsilon_{\theta_1}} - 2C_{\nabla u}; \quad b = -\frac{20\sqrt{5}}{7\sqrt{3}} Sc^{1/2} (C_{\epsilon_2} - C_{\epsilon_1}) \frac{C_{\epsilon_{\theta_3}}}{C_{\epsilon_3}};$$

$$c = C_{\epsilon_{\theta_2}} - 2.$$

This is a second-order equation for $R_\infty^{1/2}$. For the non-equilibrium terms to effectively represent residual vortex stretching in both ϵ- and ϵ_θ-equations, C_{ϵ_3} and $C_{\epsilon_{\theta_3}}$ must be positive which implies $b < 0$ since $C_{\epsilon_2} > C_{\epsilon_1}$. In addition, for consistency with current modelling, it is required that $C_{\epsilon_{\theta_2}} > 2$ i.e. $c > 0$ ($\zeta_2 < 0$). Coefficient a can be either positive or negative depending on the values of $C_{\epsilon_{\theta_1}}$ and $C_{\nabla u}$. Since $C_{\epsilon_{\theta_1}} \simeq 2$, $C_{\nabla u} < 1$ [26,27], makes a positive whereas $C_{\nabla u} > 1$ (as in Ref. [25]) implies that a is negative. As the interest of the analysis lies in retaining the effects of residual vortex stretching on both the velocity and the scalar fields, the special cases in which one constant at least among C_{ϵ_3}, $C_{\epsilon_{\theta_3}}$ and c is zero are discarded directly.

If $a > 0$, the model does not yield a physical behaviour. As a matter of fact, with $a > 0$, Eq. (9) has either no real solution or an unstable double positive node, or two positive nodes one of which, the larger one, is unstable. For initial values larger than this latter, R tends to infinity. This result is unphysical since, according to Eq. (8), $\psi_\theta \simeq aR$ for $R \to \infty$ which implies that ϵ_θ tends to zero while $\overline{\theta'^2}$ tends to a non-zero constant value.

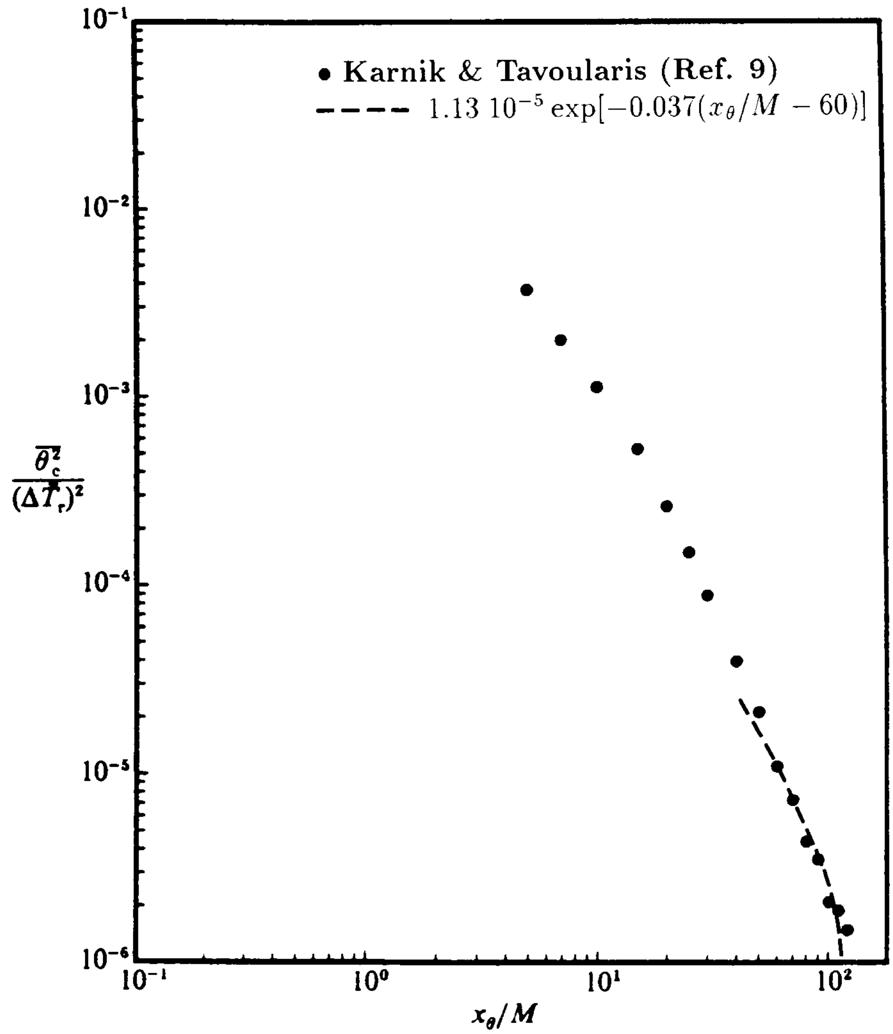

Fig. 1. Decay of normalized centreline temperature variance in the experiment of Karnik and Tavoularis (Ref. [9]).

If $a < 0$, Eq. (9) has one physical stable solution:

$$R_\infty = \left(\frac{\zeta_3}{\eta_3}\right)^2 [\Gamma(C_{\epsilon_{\theta_1}} - 2C_{\nabla u})]^{-1} \left[1 - \left(1 + \zeta_2\left(\frac{\eta_3}{\zeta_3}\right)^2 \Gamma\right)^{1/2}\right]^2 \tag{10}$$

with

$$\Gamma = \frac{C_{\epsilon_{\theta_1}} - 2C_{\nabla u}}{24Sc(C_{\epsilon_2} - C_{\epsilon_1})^2}.$$

In this case, the model produces a physical behaviour whatever the initial value of R. In Eq. (10), the molecular Schmidt number, Sc, is a characteristic of the fluid and C_{ϵ_1} and C_{ϵ_2} are standard model constants. $C_{\epsilon_{\theta_1}}$ is close to 2. and $C_{\nabla u}$ has to assume a value larger than unity as, for instance, in Ref. [25]. The equation thus comprises two undetermined parameters namely, ζ_2 and η_3/ζ_3, representing the effect of residual vortex stretching. Despite this arbitrariness, a quite interesting feature is that, within this approach, R_∞ is shown to depend on residual vortex stretching and is thereby not universal which contrasts with the result derived from standard modelling (section 3.1.).

Asymptotic forms of Eq. (10) can be obtained depending on the order of magnitude of η_3/ζ_3. From Eq. (10), it is straightforward to show that:

$$R_\infty \simeq \zeta_2(C_{\epsilon_{\theta_1}} - 2C_{\nabla u})^{-1} \quad \text{if} \quad \eta_3/\zeta_3 >> (\zeta_2\Gamma)^{-1/2}$$

and

$$R_\infty \simeq (96Sc)^{-1}\left[\zeta_2 \frac{\eta_3/\zeta_3}{(C_{\epsilon_2}-C_{\epsilon_1})}\right]^2 \quad \text{if}$$

$$\eta_3/\zeta_3 << (\zeta_2 \Gamma)^{-1/2}.$$

In the latter limit, the action exerted by mean shear on the scalar field (represented by the $C_{\nabla u}$-term of ψ_θ) is absent. This effect is retained if η_3/ζ_3 is of the same order of magnitude as $(\zeta_2\Gamma)^{-1/2}$ or larger that is, since ζ_2 is a small parameter, if $\eta_3 > \zeta_3$. This result was expectable from Eq. (8) in which the term of residual vortex stretching is proportional to ζ_3/η_3. It is also worth mentioning that the effect of residual vortex stretching is enhanced relatively to the effect of mean shear when R assumes small values since the former is represented by a $R^{1/2}$-term while, for the latter, the corresponding term is linear in R. The equilibrium scalar timescale is related to mean shear since $\tau_{\theta_\infty} = R_\infty \tau_{t_\infty}$ that is, with τ_{t_∞} derived from the model of Bernard and Speziale, $\tau_{\theta_\infty} = R_\infty/S\sqrt{C_\mu}$. In this asymptotic regime, $\psi_\theta = 2$ and $\overline{\theta'^2}$ decreases exponentially at the rate $\tau_{\theta_\infty}^{-1}$.

4. Discussion

Quantitative comparisons of model predictions with measurements are not easy for experimental data relating to the effect of uniform mean shear on an uniform fluctuating passive scalar field are apparently lacking. Sreenivasan and Tavoularis [14] have carried out experiments on zero and non-zero mean gradient temperature fields in sheared and unsheared turbulence but were specially interested in the skewness of the temperature derivatives. Other available experimental results combine the effects of mean shear and mean scalar gradients one with the other [11,12]. However, it can be conjectured that in the case of diffusion from a concentrated source, uniformity of the scalar field is approximately met on the centerline, beyond the near vicinity of the source. The measurements downstream a heated line source in an uniform shear flow by Karnik and Tavoularis [9] (specially those on the longitudinal and transversal turbulent fluxes of temperature and of its variance) suggest that the budget of temperature variance on the centerline is reduced to a convection-equals-dissipation equilibrium. This, in passing, is reminiscent of experimental results on heat diffusion from point and line sources in turbulent boundary layers [41–43]. The experimental data of Karnik and Tavoularis [9] show, in addition, that both the velocity and the scalar microscales tend to constant values from which it can be surmised that the scalar-to-velocity timescale ratio reaches an equilibrium value. It is worth mentioning that the measured integral lengthscales are not constant but increase with downstream distance. The ratio of scalar-to-velocity longitudinal integral lengthscales is found, however, to approach a constant value.

Now, the convection/dissipation regime of temperature variance together with the fact that the mechanical timescale $\tau_t = q^2/2\epsilon$ tends to a constant in this flow [11,20], imply that the R-equation takes the form $dR/dt = 1/2(\psi_\theta - 2)\tau_t^{-1}$ (Section 3.1). Hence, the constancy of R means that ψ_θ approaches 2 and that the scalar variance thereby decreases exponentially on the centerline, in the far field. Although Karnik and Tavoularis [9] fit the decay of temperature variance with a power law, it can be shown from their data that beyond $x_\theta/M = 60$, where the scalar microscale becomes constant, an exponential law of decay is also suitable. This latter is in $\exp[-\alpha(x_\theta - 60\ M)/M]$ with $\alpha \simeq 0.037$; x_θ is the distance to the heated line and $M = 0.0254$ m, the height of each channel of the flow separator. As shown in Fig. 1, the above exponential law fits reasonably well the far-field decay of temperature variance measured by Karnik and Tavoularis. The timescale of decay which, in this case, is also the scalar timescale, is: $\tau_\theta = M/\alpha U_c \simeq 0.09$ s; $U_c = 7.85$ m s^{-1} is the mean centerline velocity. Interestingly, this estimate of τ_θ is close to the value measured by Tavoularis and Corrsin [11].

The above brief qualitative analysis confirms the theoretical predictions of the models examined previously, at least in some respects namely, the asymptotic trend of λ_θ and R as well as the decay of the scalar variance. It is to be mentioned, however, that the above conclusions do not agree with the theoretical study of Majda [8] who, considering a scalar field in steady forced turbulence, showed that uniform constant mean shear drives the asymptotic decay of scalar variance to follow a power-law of time.

5. Conclusion

Under the effect of uniform mean shear and in the absence of a mean scalar gradient, spectral self-preservation analysis predicts an exponential asymptotic decay of scalar fluctuations variance with scalar microscale, integral lengthscale and scalar-to-velocity timescale ratio tending to constant values. As expected, mean shear is found to enhance the decay rate of scalar fluctuations. One-point models yield, qualitatively, a similar behaviour. To some extent, these features are corroborated by experimental data relating to heat diffusion in uniformly sheared turbulence.

Standard one-point modelling, in which q^2 and ϵ increase exponentially at large times, leads to an universal asymptotic value R_∞ of the scalar-to-velocity timescale ratio (in so far as model constants can be considered as universal). On the contrary, a model which allows for residual vortex stretching and drives the velocity field to a production-equals-dissipation equilibrium would lead to a value of R_∞ depending on residual vortex stretching in both the velocity and the scalar fields.

Estimating the validity of the different concepts is unfortunately not easy in the present case. An experimental assessment of their asymptotics would need further investigations specially, measurements conducted in a scalar field submitted to the sole effect of uniform mean shear, at large enough total strain.

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